# Dynamical effects in the fusion hindrance

David Boilley[1,2,a], Caiwan Shen[3], Yasuhisa Abe[4], and Bertrand G. Giraud[5]

[1] GANIL, CEA/DSM-CNRS/IN2P3, Bvd Henri Becquerel, 14076 Caen, France
[2] Université de Caen Basse-Normandie, F-14032 Caen Cedex, France
[3] School of Science, Huzhou Tearchers College, Huzhou, 313000 Zhejiang, P.R. China
[4] RCNP, Osaka Univ., Ibaraki, 567-0047 Osaka, Japan
[5] Institut Physique Théorique, CEA/DSM, Centre d'Etudes de Saclay, Gif-sur-Yvette, F-91191, France

**Abstract.** It is well known that there is a hindrance to fusion in collisions with heavy nuclei that plays a decisive role in synthesis of the super-heavy elements (SHE). The origin of the fusion hindrance is nowadays qualitatively understood but there are still quantitative ambiguities on the dynamics of the fusion mechanism and the predictions need to be assessed. In this communication, we stress the fact that dynamical effects play a crucial role in the amplitude of the reduction of the fusion probability. We found that the fast evolution of the neck degree of freedom affects the slow radial motion, i.e., the fusioning motion, through a dynamical coupling. We showed that we could do a so-called adiabatic elimination of the fast variable in the coupled equation, resulting in an effective one-dimensional equation for the radial motion with a shift of the starting point. This treatment of the dynamical coupling leads to a larger hindrance.

## 1 Introduction

It is well known that there is a hindrance to fusion in collisions with heavy nuclei. Since this so-called fusion hindrance plays a decisive role in the synthesis of the superheavy elements (SHE), its physical mechanism should be understood clearly. Furthermore, the fusion probability should be predicted systematically in a quantitative way.

The origin of the fusion hindrance is nowadays qualitatively understood: after crossing the Coulomb barrier, the fusing system at contact have to overcome a second barrier under strong dissipation. This inner barrier does not exist for lighter systems that directly reach the compound state after crossing the Coulomb barrier. See Fig. 1 for few examples. Such an interpretation is commonly accepted, but there are still quantitative ambiguities on the dynamics of the fusion mechanism and one has to find ways to assess the various models.

The theoretical description of the fusion is then divided into two steps: the capture process related to the crossing of the Coulomb barrier and the formation process related to the inner barrier. For the Coulomb barrier, one can easily extrapolate the models validated on the fusion of lighter nuclei. Note that for this first step a so-called fusion hindrance at energies far below the barrier also exists. In this paper, we will focus on the large hindrance at the barrier energies due to the inner barrier.

The discrepancies between the various models are twofold. Therefore, one of the challenges is to find ways to assess both the size of the inner barrier and the dynamical description of the diffusion process over it. We will briefly show our latest achievements regarding these problems in this contribution.

One of our goals is also to obtain an analytical formula of the fusion probability for an incident channel with an arbitrary combination of projectile and target nuclei.

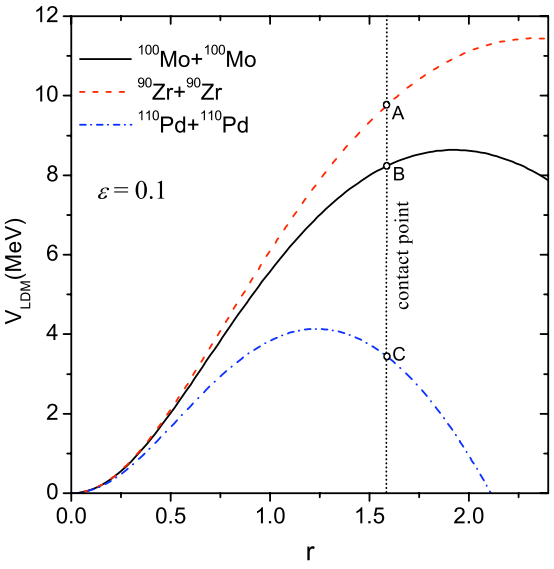


**Fig. 1.** The relation between the contact point and the saddle point for different reactions. The abscissa and ordinate stands for the distance between the two touching nuclei and LDM potential ($\varepsilon = 0.1$), respectively. The vertical dashed line corresponds to the contact point $r = 1.6$ for symmetric reactions. See text for explanation of the parameters.

[a] e-mail: boilley@ganil.fr

## 2 Appearance of the hindrance to the fusion

Very heavy nuclei have a large fissibility and become unstable if they are slightly deformed. The fission saddle point is close to the spherical shape. This means that for the reverse fusion reaction, after the contact, the fusing mononucleus faces an inner potential barrier that does not exist with light nuclei. This inner barrier is responsible for the large fusion hindrance that is observed experimentally.

To describe what happens between the touching and the compound configurations, we describe the deformed mononucleus using the so-called two-centre parameterization that involves three parameters that are the relative distance between the two centres, the asymmetry and the neck. The potential landscape is then calculated with the Liquid Drop Model.

The inner barrier that hinders fusion depends on the reaction pass followed by the mononucleus system on its journey to the compound shape. Recently, we have clarified that the neck degree of freedom of di-nucleus system is quick to reach equilibrium [1]. This is due to the fact that the potential always drives the neck towards filling the cleft between the two nuclei. This was checked systematically [2].

Up to now, the size of the neck was considered as an adjustable parameter in most of the models. The neck parameter was abitrarily fixed to 1 in Ref. [3] and 0.7 in Ref. [4]. Some other references [5] do not mention its value. Therefore, the dynamical study showing the fast denecking process is a useful step in the assessement of the various model. Note that this result contradicts the conclusion of Refs. [6,7] that argue that the neck parameter is frozen during the fusion process.

Then, we have studied systematically the appearance of the hindrance for a configuration without neck ($\varepsilon = 0.1$), for symmetric reactions [8,9] and then all reactions [2]. And a borderline was drawn between hindered and non-hindered reactions. See Fig. 2. It is validated by the experimental observations: for example, for symmetric reactions, the large hindrance phenomenon appears somewhere between the $^{100}$Mo + $^{100}$Mo and the $^{110}$Pd+$^{110}$Pd systems.

The knowledge of the location of the border between hindered and non-hindered reactions is a way to constrain the models.

## 3 Dynamical effects

### 3.1 Dissipation

The potential landscape is one of the key ingredients to understand the appearance of the hindrance to fusion. To evaluate the effect on the fusion probabilities or cross sections, one needs a dynamical study. Most of the models are based on stochastic dynamics for this step because the formation is due to the thermal diffusion over the inner barrier. They use either Langevin type equations or its equivalent Klein-Kramers one.

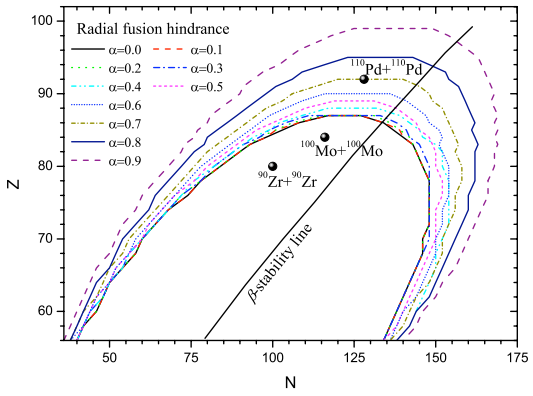


**Fig. 2.** Borderlines of the radial fusion hindrance. Reactions located inside the lines do not have radial fusion hindrance, while reactions located outside the lines are hindered in the radial direction. Three reactions and the line of $\beta$-stability are also plotted. The abscissa and ordinate stands for the neutron number $N$ and proton number $Z$ of the compound nucleus, respectively.

Assuming a simple one-dimensional parabolic barrier, the formation probability can be calculated analytically [10, 11],

$$P(K) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{B}{T}} - \frac{1}{x + \sqrt{1+x^2}}\sqrt{\frac{K}{T}}\right), \qquad (1)$$

where $B$ corresponds to the barrier height, $T$ to the temperature and $K$ to the remaining kinetic energy along the fusion path at contact. Here, $x = \beta/(2\omega)$, with $\beta$ the reduced friction and $\omega$, the angular frequency of the inverted parabola. The kinetic energy necessary to have half of the trajectories to pass over the barrier is easily calculated as

$$K_c = \left(x + \sqrt{1+x^2}\right)^2 B. \qquad (2)$$

It can be far higher than $B$, the real barrier. Dissipation appears to play an important role. As there are still ambiguities on the strength of the dissipation parameter, this introduces another parameter.

But when the dissipation is very large and the remaining kinetic energy at contact has vanished for reactions close to the barrier, this formula simply becomes

$$P(K) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{B}{T}}\right), \qquad (3)$$

in a pure diffusive regime [11]. It does not depend on $\beta$ anymore. This is due to the fluctuation-dissipation theorem: larger friction means stronger random force.

The key parameter is therefore the size of the barrier $B$ that depends on the real path followed by the mononucleus system on its journey to the compound shape. And then, the question is how to reduce a multidimensional dynamics into a one dimensional one?

### 3.2 Adiabatical approximation

The three parameters of the two-centre shell model are connected through the potential landscape, and the inertia and friction tensors.

Here, for the sake of simplicity, we only consider two degrees of freedom: the relative distance between the two centres $R$ and the neck $\varepsilon$. This will limit our analysis to symmetric reactions. The formation dynamics can be described by the two-dimension Langevin equation

$$[\gamma]\begin{bmatrix}\dot{\varepsilon}\\ \dot{r}\end{bmatrix} = -\begin{bmatrix}\partial V/\partial\varepsilon\\ \partial V/\partial r\end{bmatrix} + \begin{bmatrix}\rho_1(t)\\ \rho_2(t)\end{bmatrix}, \tag{4}$$

for which we have neglected the inertia term, in order to be consistent with the Smoluchowski approximation that corresponds to a purely diffusive process. The random force satisfies the fluctuation-dissipation theorem,

$$\langle\rho_i(t)\rho_j(t')\rangle = 2T\gamma_{ij}\delta(t-t'). \tag{5}$$

In this equation, $r$ is a dimensionless variable defined as $r = R/R_0$, $R_0$ being the radius of the compound nucleus and $\varepsilon$ corresponds to the neck variable. $\varepsilon = 1$ means two touching hard spheres and $\varepsilon = 0$ no neck.

We will assume here that around the saddle, the friction tensor $\gamma$ is independent of $r$ and $\varepsilon$. The potential map is such as it has a U shape for the neck variable and a barrier shape for the radial one. It is confining for the neck. Then, during the fast evolution of the neck, the relative distance appears to be frozen. The neck dynamics could be approximately studied as follow,

$$\dot{\varepsilon} = -[\gamma^{-1}]_{\varepsilon\varepsilon}\frac{\partial V}{\partial\varepsilon} - [\gamma^{-1}]_{r\varepsilon}\frac{\partial V}{\partial r} + r_\varepsilon(t) \tag{6}$$

$$\simeq -[\gamma^{-1}]_{\varepsilon\varepsilon}\frac{\partial V}{\partial\varepsilon} - [\gamma^{-1}]_{r\varepsilon}\left.\frac{\partial V}{\partial r}\right|_{r=r_0} + r_\varepsilon(t). \tag{7}$$

Once the neck has reached its asymptotic value, it does not evolve anymore due to the large confinement potential.

The differential equation governing the evolution of $r$,

$$\gamma_{r\varepsilon}\dot{\varepsilon} + \gamma_{rr}\dot{r} = -\frac{\partial V}{\partial r} + \rho_2(t), \tag{8}$$

should be studied on two time scales: first, during the quick evolution of the neck variable, it can be approximated by

$$\gamma_{r\varepsilon}\dot{\varepsilon} + \gamma_{rr}\dot{r} \simeq 0, \tag{9}$$

for the average value. This means that

$$\Delta r \simeq -\frac{\gamma_{r\varepsilon}}{\gamma_{rr}}\Delta\varepsilon. \tag{10}$$

Then, once the neck has reached its equilibrium, $\dot{\varepsilon} \simeq 0$ and one has

$$\gamma_{rr}\dot{r} \simeq -\frac{\partial V}{\partial r} + \rho_2(t). \tag{11}$$

This equation for the evolution of the relative distance appears to be decoupled from the neck's evolution. The effect of the fast evolution of the neck is to shift the initial value of the relative distance as given by Eq. (10).

This approximate dynamical evolution can be checked on a simple test case based on a harmonic potential that is not meant to be realistic,

$$V(\varepsilon, r) = V_s + \frac{1}{2}g\varepsilon^2 - \frac{1}{2}h(r-r_s)^2, \tag{12}$$

for which the coupled differential equations (4) can be exactly solved [10]. With such a potential, the approximate evolution of $r$ is characterized by,

$$\langle r(t) - r_s\rangle = \left((r_0 - r_s) + \frac{\gamma_{r\varepsilon}}{\gamma_{rr}}(\varepsilon_0 - \varepsilon_\infty)\right)\exp\left[\frac{ht}{\gamma_{rr}}\right] \tag{13}$$

$$\langle\delta r^2(t)\rangle = \frac{T}{h}\left(\exp\left[\frac{2ht}{\gamma_{rr}}\right] - 1\right). \tag{14}$$

Fig. 3 shows the comparison of this result with the exact solution and the uncoupled case ($\gamma_{r\varepsilon} = 0$) for the average trajectory and the fusion probability,

$$P(t) = \int_{-\infty}^{r_s}\exp\left[-\frac{(r-\langle r(t)\rangle)^2}{2\delta r^2(t)}\right]\frac{dr}{\sqrt{2\pi\delta r^2(t)}} \tag{15}$$

$$= \frac{1}{2}\mathrm{erfc}\left[\frac{\langle r(t)\rangle}{\sqrt{2\delta r^2(t)}}\right]. \tag{16}$$

The approximate solution of equations (13,14,16) agrees quite nicely with the exact solution, although $g/h = 3$ is quite weak.

It appears clearly that the fast evolution of the neck variable allows studying the evolution of the other degrees of freedom separately. In the previous sections, we showed that the potential map is very sensitive to the value of the neck. Here, we find that the dynamical coupling through the dissipation tensor shifts the effective initial value of the relative distance. See Eq. (10). After this transient regime, the fusion will then follow the path that minimizes the potential with respect to the neck and we can do an adiabatic approximation.

With this simple model, the shift is of the order of few femtometers, which is large enough to have an influence on the hindrance to the fusion. It explains the large difference between the fusion probabilities of the uncoupled case and the approximate or exact coupled case that can be observed on Fig. 3. The shift of the initial value of the relative distance is always positive in this model and enlarges the size of the barrier that has to be crossed to reach the compound shape. The fusion is then more reduced.

The effect of the shift of the effective injection point on the long time limit of the fusion probability,

$$P(t\to\infty) = \frac{1}{2}\mathrm{erfc}\left[\sqrt{\frac{V(\varepsilon_\infty, r_s) - V(\varepsilon_\infty, r_0 - r_s + \Delta r)}{T}}\right], \tag{17}$$

is the larger the heavier the system. For systems close to the hindrance border like the $^{110}$Pd+$^{110}$Pd, $r_0$ is close to the saddle and the potential is quite flat. For heavier systems, the potential has a steeper slope near the contact point.

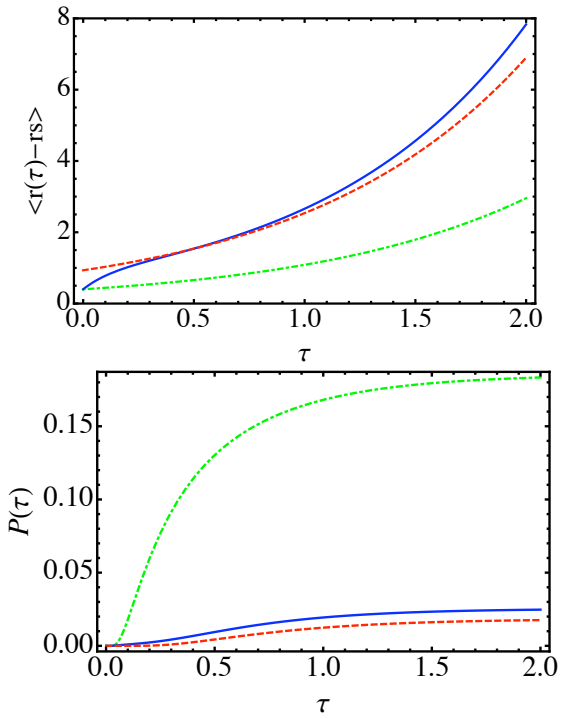


**Fig. 3.** Average trajectory (top) and fusion probability (down) as a function of time for a parabolic potential. The solid blue line represents the exact solution. The green dotted-dashed one represents the uncoupled one ($\gamma_{r\varepsilon} = 0$). The dashed red curve represents the approximate solution. See text. Here $\gamma_{\varepsilon\varepsilon}/\gamma_{rr} = 0.6$, $\gamma_{r\varepsilon}/\gamma_{rr} = 0.5$, $g/h = 3$ and $T/h = 0.2$. The time unit is $\gamma_{rr}/h$.

Therefore, the shift $\Delta r$ will cause a larger change of the potential barrier for heavier systems.

Of course, the model here is crude: the potential landscape is simple, we neglected the inertia and we assumed that the friction tensor is constant. It confirms the adiabatic approximation that is usually done in the various models, but it shows that the dynamical coupling between the neck and radial degrees of freedom induces a shift of the effective initial value of the relative distance. We will publish a full paper with a more comprehensive study on its magnitude.

Actually, in their so-called "fusion by diffusion" model, Świątecki and his collaborators [12] introduced an initial shift of the injection point considered to be an adjustable parameter ranging from 0 to 3 fm. Here, we propose a justification to it. To our knowledge, all the other models that do not explicitely include the neck dynamics do not take into account such an initial schift. In a very recent paper [13] Liu et al explore numerically the effect of the non-diagonal term of the friction tensor on the injection point. They conclude that the average injection point is not shifted. This is in contradiction with our results.

## 4 Conclusion

In this article, we have stressed the importance of the neck parameter that can change the fusion cross sections by orders of magnitude. We have shown that the neck degree of freedom evolves faster than the relative distance between the two fusing nuclei. Then the approximation of using an asymptotic value of the neck is justified.

The rapid evolution of the neck parameter changes the potential landscape seen by the other collective variables. The experimental appearance of the hindrance of the fusion for reactions with heavy nuclei confirms this conclusion. This rapid evolution of the neck also changes the initial value of the other collective variables through a dynamical coupling. For the relative distance, the shift is not negligible and should be included in the models. Our analysis gives a theoretical justification to the adjustable shift introduced by Świątecki et al [12] in order to reproduce the data. Finally, it is important to note that both effects enlarge the hindrance of the fusion.

This analysis of the influence of the neck dynamics on the fusion of heavy nuclei is mainly based on simplified analytical models and is therefore limited to symmetric reactions. The asymmetry degree of freedom complicates the analysis that cannot be simply handled with analytical toy models. Therefore, a more complete study will be published elsewhere.

Eventually, it should be noted that we have used a Markovian dynamics in this study although the characteristic time of the fusion process is such that it might not be correct [14]. This is another dynamical effect that has to be taken into account in the study of the fusion hindrance.

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